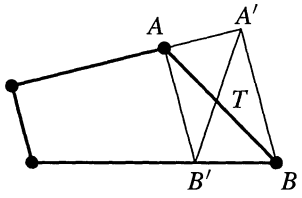
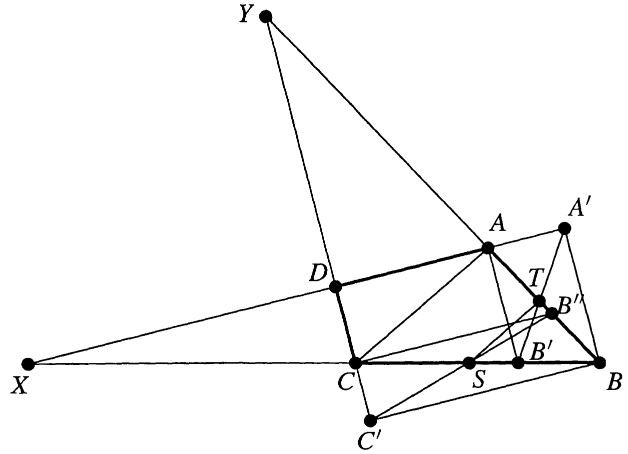
A Parallelogram Inscribed in a Quadrilateral

10810 [2000, 566]. *Proposed by Juan-Bosco Romero Márquez, Universidad de Valladolid, Valadolie, Spain.* Consider a convex quadrilateral with no parallel sides. On each side AB, select a point T as follows: Draw lines from A and B parallel to the opposite side. Let A' and B' be the new points where these lines intersect the sides neighboring AB. Let T be the point where AB intersects $A'B'$. Prove that the four points selected in this way are the corners of a parallelogram.



Solution by Mohammed Ali Salem, Zakum Development Company — CMM, Abu Dhabi, United Arab Emirates. By analogy with the construction of T, draw lines from B and C parallel to side AD. Let C' and B'' be the new points where these lines intersect lines CD and AB, respectively. Let S be the point of intersection of lines BC and $B''C'$. We prove that ST is parallel to AC.



Extend DA and CB so that they meet at X, and extend AB and DC so that they meet at Y. Similarity of $\triangle B'BA$ and $\triangle CBY$ implies that $B'B/CB = BA/BY$. Also, similarity of $\triangle B''BC$ and $\triangle ABX$ implies that $B''B/AB = BC/BX$. Thus $B'B \cdot BY = BA \cdot CB = B''B \cdot BX$, and therefore $B'B/BX = B''B/BY$. By Thales's Theorem, $B'B''$ is parallel to XY and $XB'/XB = YB''/YB$. Similarity of $\triangle XB'A$ and $\triangle XBA'$ implies that $XB'/XB = B'A/BA'$, and similarity of $\triangle YB''C$ and $\triangle YBC'$ implies that $YB''/YB = B''C/BC'$. Similarity of $\triangle B'AT$ and $\triangle A'BT$ implies that $B'A/A'B = AT/BT$, and similarity of $\triangle B''CS$ and $\triangle C'BS$ implies that $B''C/C'B = CS/BS$. Therefore $AT/TB = B'A/BA' = XB'/XB = YB''/YB = B''C/BC' = CS/SB$, $AT/TB = CS/SB$, and TS is indeed parallel to AC. The rest follows.

Solved also by J. Anglesio (France), M. Bataille (France), M. Benedicty, J. C. Binz (Switzerland), D. L. G. Bizzarri (Belgium), G. D. Brown, S. Cautis (Canada), R. J. Chapman (U. K.), B. Cheng (Philippines), P. Csiba (Slovakia), P. P. Dályay (Hungary), J. E. Dawson (Australia), D. Donini (Italy), S. B. Ekhad, J. Fukuta (Japan), P. Gao, M. Getz & D. Jones, A. Goddijn (The Netherlands), J.-P. Grivaux (France), M. Hajja (U. A. E.), L. Herot, P. Hohler (Switzerland), W. Janous (Austria), C. Koç (Turkey), N. Komanda, T. Kostic (Yugoslavia), J. H. Lindsey II, O. P. Lossers (The Netherlands), J. H. Nieto (Venezuela), A. Nijenhuis, C. R. Pranesachar (India), K. Schilling, A. Sinefakopoulos (Greece), C. Soland (Switzerland), M. Woltermann, R. L. Young, L. Zhou, Con Amore Problems Group (Denmark), GCHQ Problems Group (U. K.), and the proposer.