3221. *Proposed by Juan Bosco Romero Márquez, Universidad de Valladolid, Spain.*

Let ABC be a triangle with sides a, b, c opposite the angles A, B, C respectively. Let AH be perpendicular to the side BC with H on BC. Set $m = BH$, $n = CH$. Prove that $a(bm + cn) - bc(b + c)$ is positive, negative, or zero according as $\angle A$ is obtuse, acute or right-angled.

Solution by Francisco Javier García Capitán

If $m = BH$ and $n = CH$ (signed distances), applying the definition of cosine and cosine law to angles B and C, we have the relations $m = (c^2 + a^2 - b^2)/(2a)$ and $n = (a^2 + b^2 - c^2)/(2a)$, so that $a(bm + cn) - bc(b + c)$ can be factored as $-(b + c)(b^2 + c^2 - a^2)/2 = -(b + c)bc \cos A$ and the result follows immediately.